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TO STUDY ON WAVELETS IN OPTIMIZATION THEORY AND WAVELETS AS DIAGNOSTIC AND THERAPEUTIC TOOL

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ABSTRACT

All theoretical findings that relate to squeezing film behavior, hydrodynamics, elasto-hydrodynamics, and other phenomena can be connected to experimental findings. Numerous models of both healthy and degenerating synovial joint behavior have been published up to this point. These hypotheses do not adequately explain the complex structure and behavior of synovial joints. Numerous findings illuminating the ludicrous functioning and cause of deprivation are not empirically tested nor authenticated. Animal models are therefore the practicable model and the greatest technique to discover unanswered problems like cartilage degradation causes and conditions causing various types of arthritic diseases. Shear test, confined compression, indentation test, and other experiments have been carried out to provide a deeper understanding of the biomechanical aspect of synovial joints. The basic conclusion drawn from the results of all carried out experiments is to take preventions and precautions to the greatest extent possible to avoid replacement of joints. With particular reference to the wavelet approach, numerous techniques and models of synovial fluid's biomechanical function are comprehensively examined. Our main focus was on describing mathematical models and the methodologies used by different authors, analyzing their various features, and coming to a conclusion that would allow the bio-medical engineers to choose the proper design factors. A number of studies describing numerous fallouts are described that have assisted in obtaining a certain result, advancing our understanding of the clinical and diagnostic approaches to synovial joints. To acquire a better and deeper grasp of the synovial joints' complex nature as well as a clear picture of it, advantages and disadvantages are also highlighted. The intricacy of the construction and operation of the synovial joint can be better understood by looking at certain metrics, such as load carrying capacity, frictional force, and sliding velocity.

KEY WORDS: *Wavelets, Optimization, Theory, Techniques, strategies.*

INTRODUCTION

THEORETICAL DISCOVERIES IN CONTRAST WITH INVESTIGATIONAL OUTCOMES

The research standout feature is how it discusses the value and applicability of wavelets and wavelet packets in the analysis of a modified Reynolds equation that governs knee joint performance. The current effort's main findings and the effects they have in the real world are accessible for the technical community's benefit, establishing its justification. We can compare experimental data with classical ideas thanks to research that uses lubrication theory in relation to synovial joints and mathematical models that incorporate experimental results. Theoretically, the solution of the Reynolds equation, which describes pressure distribution within these joints, is crucial in determining specific characteristics, such as ideal load carrying capacity, reduced friction, and longer-lasting skeleton bearings. These findings have assisted us in our investigation of certain biomechanical characteristics of connections between synovial fluid, cartilage, and joint bones. To explain various characteristics including sliding speeds, friction, and load bearing capacity, several simple synovial joint functions are recorded experimentally. When the human body is subjected to a sufficient to high load, it has been demonstrated through the difference between theoretical and experimental results that typical synovial joints exhibit modest sliding speed and little friction.

APPLICATION OF WAVELETS IN OPTIMIZATION THEORY

The standard optimization method uses an objective function, a continuously differentiable function, to discover maxima or minima. Initial objective function is offered with or without constraints during the process. Because the objective function may or may not satisfy the two characteristics, namely continuous and differentiability, these processes have very little chance of being used in real-world applications. As a result, a variety of numerical algorithms have emerged, some of which use maximum-type objective functions that may or may not contain constraints. These optimization methods need a lot of time and extensive computation. Since wavelet theory has evolved, a number of strategies built on improvements in the mathematical underpinnings of wavelet theory have demonstrated that better results in terms of time, expense, and accuracy can be attained. There are numerous papers that discuss the advantages of various methods that use various wavelet-based strategies. The same motivation and goal behind this chapter's compilation and review of some wavelet-based techniques, which also highlights their benefits and drawbacks when compared to more conventional approaches. The applications of wavelets in optimization theory and optimum control issues are briefly discussed as a byproduct.

In a wide range of problems connected to various criteria that may vary from objective to outcome, multiple techniques are needed. In order to provide better and faster results, these types of problems are tackled utilizing mathematical modeling in a wavelet environment. Modeling an optimization problem typically involves the following steps: Identifying the need for optimization, choosing design variables, mentioning constraints pertaining to the optimal solution, designing an objective function, setting up variable boundaries, and selecting an optimization algorithm to find the solution, if and when it exists. Calculus-based approach and enumerative method are two categories that can be used to classify optimization algorithms. Calculus-based optimization techniques use gradient focused searching to reduce the error surface of the objective function. Local goals are typically attained for objective functions that are unclear or multimodal. The goal of signal processing stays the same because there aren't many possibilities, such as noisy, fuzzy, imprecise, or discontinuous signals.

More storage capacity is needed to store data for signal processing, video compression, and image compression for transmission, processing, gaining, and retrieval due to several ongoing improvements in communication technology. Due to this restriction, specimen processing and optimization approaches were successfully used to conserve loading space, reduce complexity, minimize data processing, and ultimately lessen overall procedure costs. Visual observations have played a key part in defining knowledge from the dawn of invention and research, contributing to an advance in the standard of living. With the aid of science, mathematics, and technological innovation, it is now more likely than ever to process visual data effectively. Numerous algorithms are created for this goal, together with optimization theory, to achieve the necessary technical improvement in image and video processing (Agarwal et al., 2017)

WAVELETS AS DIAGNOSTIC AND THERAPEUTIC TOOL FORKNEE JOINT PROBLEMS

The chapter's primary goal is to provide a general overview of the biomechanical characteristics of synovial joints. This has been included in the inquiry work done as part of the wavelets application research topic. It is generally widely accepted that every second individual experiences joint problems. Although there are many distinct types of joints in human bodies, synovial joints are of primary relevance because they can be thought of as joints for quality of life. The knee joint, out of all the synovial joints, serves two functions at once: mobility and load bearing capacity (the ability to support both the body's weight and additional weight, if necessary).

The knee joint in the human body is one of the biggest and most intricate joints. Everything functions normally and the intricate network of the human knee joint coordinates with the rest of the body when there are no diseases or deficiencies. However, medical treatment with or without surgery becomes the only viable option for repair if a disease or injury impacts the functionality of any network of bodily parts. In general, the human body's most active joints are thought to be synovial joints, notably the knee, which is replaced in extremely rare circumstances when it is injured or ill. Arthritis is a serious condition that affects the joints. There are various forms of arthritis and ailments that are connected to it.

Arthritis can be painful and restrict a person's ability to conduct daily tasks in people of all ages and sexual orientations. The knee and hip joints can carry or drag heavy objects since they can support our complete body weight. These joints feature cartilage, which acts as a cushioning surface, at the ends of the joint bones. This cartilage begins to deteriorate over time, which causes cysts and bony spurs to form at the ends of the joint bones. Osteoarthritis is the term for this form of degenerative arthritis (OA). Rheumatoid arthritis is a different variety of arthritis (RA). As an autoimmune condition, RA causes our immune system to wrongly target the soft tissue around our joints as a virus or bacteria, eventually leading to bone deformities.

Synovial joints are designed by nature to last as long as possible with minimal wear and friction while performing their intended function. Synovial fluid therefore plays a significant role in the field of study. The two main objectives of synovial joint research are as follows: First, consider the fundamentals of the natural lubricating process. Second, creating and using artificial joints that are useful for replacing joints The ability to predict answers to common problems that are challenging to measure empirically has been facilitated by mathematical models.

Many scientists and researchers are focusing on the bio-mechanics of the lubrication of the knee joint so that sophisticated models can be used to do complex numerical computations that can produce the required results. Wavelets and models based on wavelets, which are the best component because of their organized structure and localization feature, have been used extensively in biomedical engineering.

BIOPHYSICAL, CHEMICAL AND MECHANICAL ASPECTSOF SYNOVIAL JOINTS

Physically, a synovial joint is made up with two connecting bones, with cartilage and synovial fluid filling the spaces between them. About 0.2 to 0.5 ml of colorless or occasionally slightly yellowish synovial fluid was discovered. The chemical composition of synovial fluid includes some protein plasma, which contributes to its rheological properties.

Numerous scientists and researchers have investigated various material constants and functions used in the rheological fluid model that is suggested for lubricating synovial fluid. Higher grade fluid was modeled by Nigam et al. (1983) as a lubricant in synovial joints.

They arrived at the analytical expression for the pressure distribution and load carrying capacity. Young synovial fluid typically exerts greater pressure on the joint cavity than does synovial fluid from elderly or osteoarthritic individuals. The center and at exists showed the least noticeable pressure distribution. The intermediate portions were where it was most visible. When compared to normal, old, and OA synovial fluid, the two surfaces of the bones move toward one another more slowly with youthful synovial fluid.

It is believed that the spherical surface of the bones at the exit approaches the plane surface when two bones approach one another in the absence or minimization of synovial fluid. Standing and jumping positions increase the amount of time that these two synovial joint bones are in touch with one another. Wear-related severe cartilage structural breakdown results from this.

Let's now talk about the biomechanical side of synovial joints, which is muscle action. Human movement is viewed as a series of cause-and-effect events. Studies on human walking have been conducted, along with the publication of numerous articles and theories.

The four primary components of the knee are: bones, ligaments, cartilage, and tendons. Three bones make up the knee joint: the patella (kneecap), the tibia (shinbone), and the femur (thigh bone). Ligaments are used to connect bones to other bones.

There are four different kinds of ligaments that keep the bones firmly in place and give the knee its stable position. Two major cruciate ligaments, out of a total of four ligaments, are located inside the knee joint. Anterior Cruciate Ligament (ACL) and Posterior Cruciate Ligament are their names (PCL). These two ligaments come together to form an X, with the anterior cruciate coming from the front and the posterior from the back. The following categories best describe how the human knee joint functions biomechanically:

1. The ACR runs diagonally from the center section and prevents both the tibia and femur from sliding forward and backward at the same time.
2. The PCL ligament is positioned far inside the knee joint. Additionally, it is separated from the fluid-filled synovial cavity and travels down diagonally. It stops the tibia from moving posterior to the femur and the

femur from sliding rearward on the anterior margin of the tibia.

3. Medial and lateral collateral ligaments, the other two ligaments, prevent the femur from slipping to the side.

The knee's smooth, pain-free performance is made possible by the knee cartilage. Articular cartilage and meniscus cartilage are the two forms of cartilage that can be found inside the knee. The smooth covering above joints that protects the ends of bones is called articular cartilage. Meniscus cartilage can be seen as C-shaped spongy pads between the tibia and femur. The menisci act as a shock absorber and stabilizer since they are formed of thick, rubbery cartilage. The knee joint contains two different types of menisci. One is the lateral menisci, which are located outside the knee, and the other is the medial menisci, which are located inside the knee joint. The knee's smooth operation is supported by a number of synovial fluid-filled sacs. Let $L = L(t)$ be the force that begins to act when a person stands stationary or begins to move. The femoral condyles and tibia plateau are compressed when $L > 0$ and the squeeze film action is started at L 0. This suggests that fluid in the knee joint either drains into or is pulled out of the cartilage. Due to its lack of a blood supply, cartilage is fed by synovial fluid. It has been observed that when a person is immobile in the standing position for an extended period of time, synovial fluid is forced out of the space between the tibial plateau and the femoral condyles, causing a straight connection of the surfaces of the cartilage that covers the bones.

MATHEMATICAL MODELLING OF SYNOVIAL FLUID FLOW

A tool that has proven useful in describing a variety of phenomena that pertain to physical, biological, and engineering are just a few examples is mathematical modeling. The modeling procedure typically results in a mathematical situation where the system represents an equation, such as an ordinary or partial differential equation. For the purposes of the mathematical study, the knee joint is modeled as a cylinder with a plane configuration. It is the most practical method for simulating and analyzing these kinds of synovial joints. All of a solid's properties, like as rigidity, apply to the bones that make up these joints. Articular cartilage, which has a porous layer and elastic-like material, separates these joints in the human knee. The synovial fluid, a Newtonian lubricant, fills the space between these joints. Reynold's equation is used to quantify the pressure the human body exerts on the synovial fluid by considering surface structure, angles, and velocities:

The following is a modified Reynolds equation for the conventional lubrication theory:

$$\frac{\partial}{\partial x}\left\{\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right\}+\frac{\partial}{\partial y}\left\{\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial y}\right\}=\frac{\partial}{\partial x}\left\{\frac{\rho h(u_a+u_b)}{2}\right\}+\frac{\partial}{\partial y}\left\{\frac{\rho h(v_a+v_b)}{2}\right\}+\rho(w_a-w_b)-\rho u_a\frac{\partial h}{\partial x}-\rho v_a\frac{\partial h}{\partial y}+h\frac{\partial p}{\partial t}$$

The following parameters and notations are used in the equation above:

- Synovial fluid film pressure: p
- Cartesian coordinates are used for measurements like bearing length and width.

- The synovial fluid film thickness is h, the synovial fluid viscosity is, the synovial fluid density is, and the bounding body's velocities in the directions of x, y, and z are, respectively, u, v, and w.

- Equations with subscripts a and b show the top and bottom bounding bodies, respectively.

When revising Reynolds' equation, boundary conditions such as p = 0 at the inlet and p = 6p = 0 at the outflow were taken into account. The smooth surface, Newtonian fluid, iso-viscous fluid, steady state condition, incompressible fluid, disregard for the inertia effect, and isothermal condition were among the six assumptions taken into account for the solution of Reynold's equation. These presumptions aided in the modeling of the pressure acting on the lubricant, leading to the development of a modified Reynold's equation.

The average flow model, finite element approach, and finite difference method have all been used to solve this equation using various assumptions and approximations. The modified Reynold's equation was first calculated to determine the effect of pressure on a rough surface when bones are partially lubricated. After then, numerous adjustments were made and numerous estimations were calculated.

TOTAL KNEE REPLACEMENT AS AN ULTIMATE OPTION

Human joint replacements have been done for a few decades now. The use of the knee joint as a therapy for arthritis has been rising. Since artificial joints can support loads, they should be highly resistant to wear and tear. Metal-on-plastic bearings were previously utilized, however plastic only wore down an average of 0.13mm annually. Metal over plastic was adopted as a result, although it was discovered that metal-on-metal procedures had far less wear than plastic. Metal on metal bearings were now more suited for long-term use as a result. Through a variety of studies utilizing various lubricants, several metals and their wear properties were studied. In order to determine the chemical composition and wear progression, the lubricants used in the tests were compared with synovial fluid.

In an experiment, Vitallium metal and synovial fluid were used to compare friction and the progression of wear. Vitallium metal is a cobalt and chromium alloy with corrosion and thermal resistance properties. This characteristic makes Vitallium a popular material for knee prosthesis. Other lubricants were utilized in the experiment to study the friction between metal movements in place of synovial fluid. According to the results of this experiment, the protein composition of synovial fluid is the optimum component for lubricating boundaries and forming iconic bonds that lessen friction.

Early arthritis diagnosis is challenging, in part because the overlap of normal and diseased joints is not properly studied. The essential component for identifying the stages of arthritis is synovial fluid. A generalized investigation of its volume, color, viscosity, and clarity provides a generalized picture of arthritis. Cui presented a unique approach that uses the fuzzy C-means cluster adaptive wavelet (FCMC-AW) algorithm because the process for making a definitive diagnosis can take several months. This technique improved the automated pattern recognition application (APR).

This procedure has advanced in two ways: using the adaptive wavelet method-based M-band feature extractor first. Following that is a Bayesian classifier that takes into account the data from the FCMC-generated membership matrix. This algorithm's primary function, which sets it apart, is the rigorous classification of newly discovered patterns. This algorithm's ability to provide a classification confidence value was a key benefit.

From a clinical standpoint, radiography, biomarker analysis, genetic technologies, and other methods are available to track the evolution of knee disease.

The number of hip and knee replacements has been steadily rising over the last few decades. Historically, the most common treatment method has been surgically removing or replacing the damaged joint tissue. There are, however, few treatments that can halt or stop the process. Peri-articular osteotomy and femoro-acetabular impingement are two possible therapies for slower or reverse processes. Both treatments are reasonably priced and completely rely on early detection. While arthroplasty is one of the effective procedures for reducing pain and improving function, it has disadvantages compared to treatments that keep the joint intact. Drugs that have the potential to change disease progression fall short of Food and Drug Administration FDA requirements. Treatment for OA patients will benefit from sophisticated, sensitive, and reliable techniques that can detect early OA and show how it develops.

This would lead to a decline in the number of patients needing arthroplasty. Techniques and many trials can help subjects who are consistently impacted by it.

The need for early monitoring of bone and cartilage degeneration has arisen with the emergence of new OA treatment procedures. Brown therefore conducted a review in 2009 that emphasizes the difficulty in developing an efficient assessment technique. It also sheds information on developments in arthroscopy instruments. Distinguishing damaged tissue from various normal tissues was raised as a major problem. Although this problem has been discussed extensively and has received a number of remedies, it is made worse by the challenge of connecting physical characteristics like mobility and pain to structural characteristics. As these difficulties were addressed, other ones emerged, such as the need for quantitative tissue evaluation in addition to chemical, functional, and structural approaches.

RECENT ADVANCED SCHEMES IN TREATMENT OF SYNOVIALJOINT ABNORMALITIES

Due to cartilage loss, which causes friction between the joint bones and causes discomfort and inflammation, OA typically affects elderly persons. Imaging, blood tests, and ultrasound are used to identify cartilage degeneration. Early diagnosis allows for timely management of the disease's negative effects. Acoustic emission (AE), a non-invasive and painless method, has recently been employed to diagnose the illness in a real-time system. When a material is under pressure or load, an elastic wave called an acoustic emission is produced. Sarillee et al. released a paper in 2014 that details the state of the knee joint for those with normal knees and those with OA utilizing AE sensors. In the experiment that included both the normal and OA conditions, few participants participated. In the study, an AE wave was produced after compressed bone was exposed to external stress. The resistance between the cartilages was caused by the applied external force. To record the AE signals, a few protocols were established for certain straightforward movements like sit-stand-sit and leg swings. Following a decomposition of these signals, Daubechies 4 was used as a mother wavelet in WPT analysis. The skewness and kurtosis of each decomposed signal were subsequently determined. Reduced dimension of retrieved features were classified before and after Principal Component Analysis (PCA) using support vector machines (SVM) and feed-forward neural networks (FFNN). The outcome shows that the mean frequency for straightforward activities is highest after PCA.

The relationship between the underlying forces in the lower leg and the activity of the muscles has been simulated by cutting-edge ideas in optimization. Francisco et al. developed a novel method using artificial neural networks in which they link EMG to joint motions and joint angles. 16 muscles from the input vectors were mapped onto the output vector of joint motions and angles at the knee, hip, and ankle during this mapping process. The results of this experiment support the model and imply that neural networks have the greatest potential and are the best platform for imitation in the field of biomedical engineering.

CONCLUSION

Some of the techniques use wavelets that have been technically improved and are based on them. In order to deploy technologies and achieve the needed precision, several accessible wavelets are adjusted along with their mother wavelets and characteristics. These methods can be used in a variety of disciplines, including computer science, mechanical, electronic, and biomedical engineering. This chapter's main goal is to illustrate a few real-world examples of optimization strategies that use wavelet-based approaches from various domains.

The cartilage surface becomes more vulnerable to wear during irregular gait. This occurs as a result of wear particles that eventually obstruct knee motion because they are encapsulated in synovial fluid. Based on the surface shape of these particles, classification is possible. Typically, a preset class linked with a wear process, such as cutting or rubbing, is assigned to wear particles. As a result, classification is accomplished in two steps: First, it is characterized based on variables such as periodicity, coarseness, etc. The following step is to group these variables under a predetermined class based on these determined factors.

Parameter identification is problematic since the values of these parameters are not constant and change with the position, scale, and angle of the data being obtained. There are many methods for selecting parameters, including feature combinations, exhaustive searches, and step-by-step selection. However, none of the methods provide accurate results that aid in class distinction. In addition to this laborious procedure of choosing huge parameters, calculation also takes a lot of time and effort. As a result, classes were determined by the parameters with the lowest measure of dissimilarity. Understanding the root reason, regularly tracking progress, classifying wear, and avoiding failures have all benefited from this. In response to the evolving surface morphology of wear particles, Podsiadlo et al. presented an article that categorizes wear particles using the dissimilarity measure.

Ushenko carried out another experiment to create an ocular model of synovial fluid in a human joint. From healthy patients and arthritic patients, an optically thin layer of synovial fluid was taken into account for the experimental data. Researchers looked at images of the layers of synovial fluid from various people, showing how accessible two fractions—optically isotropic and liquid crystal network—are. Statistical moments, correlation, and self-similar organization of polarization in dual dimensional distributions were some of the characteristics of synovial fluid that were examined. Objective standards for early diagnosis have also been established, and these include variances in synovial fluid pathological alterations and the degree of wavelet constant self-similarity on various scales.

Rheumatoid arthritis harms bodily joints and other physiological tissues in an irreversible manner. The Metacarpophalangeal (MCP) joints are one of the areas of our body that experience changes. We can see these changes thanks to high frequency ultrasound. Identification of the MCP capsule region with precise segmentation of the bone surface is the main hurdle that ultrasonic image processing must overcome in this process. Consequently, an algorithm was developed to deal with this problem. Using the log Gabor filter to remove noise from the photos helped the algorithm extract ridge-like structure from the images. When compared to other bone sections, experimental results in terms of mean pixel error show an acceptable value.

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